

On integrability of massless $AdS_4 \times \mathbb{CP}^3$ superparticle equations

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Abstract

Lax representation is elaborated for the equations of motion of massless superparticle on the $AdS_4 \times \mathbb{CP}^3$ superbackground that proves their classical integrability.

1 Introduction

Discovery of the integrability of $AdS_5 \times S^5$ superstring equations [1] and of the dilatation operator in the dual $D = 4$ $\mathcal{N} = 4$ super-Yang-Mills theory [2] determined the direction of major progress in studying the AdS_5/CFT_4 correspondence [3] over the last decade.² The situation with the Aharony-Bergman-Jafferis-Maldacena (ABJM) correspondence [5] is more intricate. Conjectured gravity dual of $D = 3$ $\mathcal{N} = 6$ superconformal $U(N)_k \times U(N)_{-k}$ gauge theory reduces to a string theory only in the special sublimit of the 't Hooft limit defined by the conditions $k^5 \gg N \gg 1$. Moreover this IIA superstring theory 'lives' on $AdS_4 \times \mathbb{CP}^3$ superbackground [6] preserving 24 of 32 space-time supersymmetries contrary to the IIB $AdS_5 \times S^5$ superbackground that is maximally supersymmetric. Non-maximal supersymmetry on both sides of ABJM duality significantly complicates its exploration (see [7] for a review).

Thus application of the supercoset approach, suggested in [8] to construct the $AdS_5 \times S^5$ superstring action as a $2d$ sigma-model on $PSU(2,2|4)/(SO(1,4) \times SO(5))$ supercoset manifold, gives only part of the $AdS_4 \times \mathbb{CP}^3$ superstring action since only a $(10|24)$ -dimensional subspace of the $AdS_4 \times \mathbb{CP}^3$ superspace is isomorphic to the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset manifold [9], [10].³ These are 24 Grassmann coordinates of the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset manifold that are in one-to-one correspondence with the supersymmetries of the $AdS_4 \times \mathbb{CP}^3$ superbackground so that construction of the complete $AdS_4 \times \mathbb{CP}^3$ superstring action [12] requires extension of the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model by 8 fermionic fields associated with the broken supersymmetries.⁴ To this end more roundabout way is to be taken that relies on double-dimensional reduction [13], [14] of the $D = 11$ supermembrane on $AdS_4 \times S^7$ superbackground [15]. Since this background is maximally supersymmetric the supermembrane action is constructed using the generalization of the supercoset approach [8]. Another necessary ingredient is the Hopf fibration realization of the 7-sphere $S^7 = \mathbb{CP}^3 \times S^1$ [16], [17] whose S^1 fiber is identified with the world-volume compact dimension in the process of double-dimensional reduction. Resultant $AdS_4 \times \mathbb{CP}^3$ superstring action has rather complicated

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²For a recent collection of reviews see [4].

³Alternative approach to constructing the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model action relies on the introduction of the pure spinor variables [11].

⁴The $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model arises upon partial κ -symmetry gauge fixing of the $AdS_4 \times \mathbb{CP}^3$ superstring by setting to zero 8 Grassmann coordinates related to broken supersymmetries. This constrains its range of application [9], [12].

highly non-linear structure that does not simplify enough even after fixing the κ -symmetry gauge [18], [19] to address directly issues of quantization and spectrum identification.

At the same time $PSU(2, 2|4)/(SO(1, 4) \times SO(5))$ and $OSp(4|6)/(SO(1, 3) \times U(3))$ sigma-models are known to belong to a class of classically integrable $2d$ sigma-models on supercoset manifolds with \mathbb{Z}_4 -graded isometry superalgebras [20], [21], [22] so one may hope to establish integrability of the $AdS_4 \times \mathbb{CP}^3$ superstring equations following from the complete action by properly extending the Lax representation of the $OSp(4|6)/(SO(1, 3) \times U(3))$ sigma-model equations [9], [10] by contributions of 8 'broken' fermions. The first steps in this direction were made in [23], [24] where the $OSp(4|6)/(SO(1, 3) \times U(3))$ sigma-model Lax connection was extended by linear and quadratic terms in the 'broken' fermions in such a way that its curvature turns to zero up to quadratic order in them on the superstring equations truncated to the same order. Even at this order the expression for the extended Lax connection appears rather involved. That is why in [25] we suggested to use the κ -symmetry gauge freedom to retain in the sector of broken supersymmetries a part of coordinates the complete expansion in which of the Lax connection can be recovered with less efforts and then by gradually relaxing the gauge condition the dependence on other coordinates may be examined.

To simplify the problem as much as possible one can initially concentrate on the zero-mode sector described by the massless superparticle model on the $AdS_4 \times \mathbb{CP}^3$ superbackground. In Ref. [26] after working out the Lax representation for equations of motion of the massless $OSp(4|6)/(SO(1, 3) \times U(3))$ superparticle [10] we extended it to include contributions of 4 Grassmann coordinates related to the broken part of $D = 3$ $\mathcal{N} = 8$ Poincare supersymmetry while other 4 coordinates related to the broken part of conformal supersymmetry were gauged away. Here we relax this partial κ -symmetry gauge condition and prove integrability of the full-fledged $AdS_4 \times \mathbb{CP}^3$ superparticle equations. Namely we show that they admit the Lax representation

$$\frac{d\mathcal{L}}{d\tau} + [\mathcal{M}, \mathcal{L}] = 0, \quad (1)$$

where \mathcal{M} is the world-line projection of the left-invariant $osp(4|6)$ Cartan forms definition and \mathcal{L} can be presented as the $osp(4|6)$ superalgebra valued differential operator acting on the superparticle action functional

$$\begin{aligned} \mathcal{L} = & \left(M_{0'm} \frac{\partial}{\partial G_{\tau 0'm}} + \frac{1}{2} D \frac{\partial}{\partial \Delta_{\tau}} - M_{mn} \frac{\partial}{\partial G_{\tau mn}} - M_{3m} \frac{\partial}{\partial G_{\tau 3m}} + T_a \frac{\partial}{\partial \Omega_{\tau a}} + T^a \frac{\partial}{\partial \Omega_{\tau}^a} \right. \\ & \left. + \tilde{V}_a^a \frac{\partial}{\partial \tilde{\Omega}_{\tau b}^b} - \frac{1}{4} Q_{(1)\mu}^a \frac{\partial}{\partial \tilde{\omega}_{(3)\tau\mu}^a} + \frac{1}{4} \bar{Q}_{(1)\mu a} \frac{\partial}{\partial \omega_{(3)\tau\mu a}} + \frac{1}{4} Q_{(3)\mu}^a \frac{\partial}{\partial \tilde{\omega}_{(1)\tau\mu}^a} - \frac{1}{4} \bar{Q}_{(3)\mu a} \frac{\partial}{\partial \omega_{(1)\tau\mu a}} \right) S \end{aligned} \quad (2)$$

as was proposed in [26].

Organization of the paper is the following. In Section 2 necessary data on the geometry of $AdS_4 \times \mathbb{CP}^3$ superspace is briefly reviewed and then in Section 3 equations of motion for the $AdS_4 \times \mathbb{CP}^3$ superparticle are derived and their Lax representation is worked out.

2 $osp(4|6)$, $osp(4|8)$ Cartan forms and $AdS_4 \times \mathbb{CP}^3$ supervielbein

Expressions for the geometric constituents of the $AdS_4 \times \mathbb{CP}^3$ superspace were obtained in Ref. [12]. In this Section to make the presentation self-contained we review the construction of the supervielbein bosonic components putting emphasis on the realization of $osp(4|6)$ and

$osp(4|8)$ isometry superalgebras of $AdS_4 \times \mathbb{CP}^3$ and $AdS_4 \times S^7$ superbackgrounds as $D = 3$ $\mathcal{N} = 6, 8$ superconformal algebras of dual ABJM gauge theory [5].

Left-invariant $osp(4|6)$ Cartan forms in the conformal basis admit the following decomposition [27]

$$\begin{aligned}\mathcal{C}(d) = \mathcal{G}^{-1}d\mathcal{G} = & \Delta(d)D + \omega^m(d)P_m + c^m(d)K_m + G^{mn}(d)M_{mn} \\ & + \Omega_a(d)T^a + \Omega^a(d)T_a + \tilde{\Omega}_a{}^b(d)\tilde{V}_b{}^a + \tilde{\Omega}_b{}^a(d)\tilde{V}_a{}^b \\ & + \omega_a^\mu(d)Q_\mu^a + \bar{\omega}^{\mu a}(d)\bar{Q}_{\mu a} + \chi_{\mu a}(d)S^{\mu a} + \bar{\chi}_\mu^a(d)\bar{S}_a^\mu\end{aligned}\quad (3)$$

over the generators of $D = 3$ conformal algebra (D, P_m, K_m, M_{mn}) , $su(4) \sim so(6)$ generators divided into the generators $\tilde{V}_b{}^a$ of the $U(3)$ stability group of $\mathbb{CP}^3 = SU(4)/U(3)$ manifold and the coset generators (T_a, T^a) , and fermionic generators of $D = 3$ $\mathcal{N} = 6$ Poincare $(Q_\mu^a, \bar{Q}_{\mu a})$ and conformal $(S^{\mu a}, \bar{S}_a^\mu)$ supersymmetries that carry $SL(2, \mathbb{R})$ spinor index $\mu = 1, 2$ and $SU(3)$ (anti)fundamental representation index $a = 1, 2, 3$ in accordance with the decomposition $\mathbf{6} = \mathbf{3} \oplus \bar{\mathbf{3}}$ of the $SO(6)$ vector representation on $SU(3)$ representations.

To characterize the geometry of $OSp(4|6)/(SO(1, 3) \times U(3))$ supermanifold Cartan forms related to the generators $g_{(k)}$ with definite eigenvalues i^k under the \mathbb{Z}_4 automorphism of the $osp(4|6)$ superalgebra are used. Corresponding form of (3) is

$$\mathcal{C}(d) = \mathcal{C}_{(0)}(d) + \mathcal{C}_{(2)}(d) + \mathcal{C}_{(1)}(d) + \mathcal{C}_{(3)}(d), \quad (4)$$

where

$$\begin{aligned}\mathcal{C}_{(0)}(d) &= 2G^{3m}(d)M_{3m} + G^{mn}(d)M_{mn} + \tilde{\Omega}_a{}^b(d)\tilde{V}_b{}^a + \tilde{\Omega}_a{}^a(d)\tilde{V}_b{}^b \in g_{(0)}, \\ \mathcal{C}_{(2)}(d) &= 2G^{0'm}(d)M_{0'm} + \Delta(d)D + \Omega_a(d)T^a + \Omega^a(d)T_a \in g_{(2)}, \\ \mathcal{C}_{(1)}(d) &= \omega_{(1)a}^\mu(d)Q_{(1)\mu}^a + \bar{\omega}_{(1)}^{\mu a}(d)\bar{Q}_{(1)\mu a} \in g_{(1)}, \\ \mathcal{C}_{(3)}(d) &= \omega_{(3)a}^\mu(d)Q_{(3)\mu}^a + \bar{\omega}_{(3)}^{\mu a}(d)\bar{Q}_{(3)\mu a} \in g_{(3)}.\end{aligned}\quad (5)$$

Definite eigenvalues under the \mathbb{Z}_4 isomorphism have the $so(2, 3)$ generators

$$M_{0'm} = \frac{1}{2}(P_m + K_m), \quad M_{0'3} = -D, \quad M_{3m} = \frac{1}{2}(K_m - P_m) \quad (6)$$

and fermionic generators

$$Q_{(1)\mu}^a = Q_\mu^a + iS_\mu^a, \quad \bar{Q}_{(1)\mu a} = \bar{Q}_{\mu a} - i\bar{S}_{\mu a}; \quad Q_{(3)\mu}^a = Q_\mu^a - iS_\mu^a, \quad \bar{Q}_{(3)\mu a} = \bar{Q}_{\mu a} + i\bar{S}_{\mu a}. \quad (7)$$

Bosonic and fermionic Cartan forms from $\mathcal{C}_{(1,2,3)}$ eigenspaces

$$G^{0'm}(d) = \frac{1}{2}(\omega^m(d) + c^m(d)), \quad \Delta(d), \quad \Omega_a(d), \quad \Omega^a(d) \quad (8)$$

and

$$\omega_{(1)a}^\mu(d) = \frac{1}{2}(\omega_a^\mu(d) + i\chi_a^\mu(d)), \quad \omega_{(3)a}^\mu(d) = \frac{1}{2}(\omega_a^\mu(d) - i\chi_a^\mu(d)) \quad (9)$$

and c.c. are identified with the $OSp(4|6)/(SO(1, 3) \times U(3))$ supervielbein bosonic and fermionic components, other Cartan forms

$$G^{3m}(d) = -\frac{1}{2}(\omega^m(d) - c^m(d)), \quad G^{mn}(d), \quad \tilde{\Omega}_a{}^b(d) \quad (10)$$

describe the $SO(1, 3) \times U(3)$ connection.

Geometric constituents of the $OSp(4|8)/(SO(1,3) \times SO(7))$ supermanifold are constructed out of the $osp(4|8)$ Cartan forms that in conformal basis read [19]

$$\begin{aligned}\hat{\mathcal{G}}^{-1}d\hat{\mathcal{G}} = & \underline{\Delta}(d)D + \underline{\omega}^m(d)P_m + \underline{c}^m(d)K_m + \underline{G}^{mn}(d)M_{mn} \\ & + \Omega_a(d)T^a + \Omega^a(d)T_a + \tilde{\Omega}_a(d)\tilde{T}^a + \tilde{\Omega}^a(d)\tilde{T}_a \\ & + \tilde{\Omega}_a{}^b(d)\tilde{V}_b{}^a + \tilde{\Omega}_b{}^a(d)\tilde{V}_a{}^b + \Omega_a{}^4(d)V_4{}^a + \Omega_4{}^a(d)V_a{}^4 + h(d)H \\ & + \underline{\omega}_\mu^\mu(d)Q_\mu^\mu + \underline{\omega}^{\mu a}(d)\bar{Q}_{\mu a} + \underline{\chi}_{\mu a}(d)S^{\mu a} + \underline{\bar{\chi}}_\mu^a(d)\bar{S}_a^\mu \\ & + \omega_4^\mu(d)Q_\mu^4 + \bar{\omega}^{\mu 4}(d)\bar{Q}_{\mu 4} + \chi_{\mu 4}(d)S^{\mu 4} + \bar{\chi}_\mu^4(d)\bar{S}_4^\mu.\end{aligned}\tag{11}$$

There can be chosen $OSp(4|8)/(SO(1,3) \times SO(7))$ representative suitable for the reduction to 10 dimensions

$$\hat{\mathcal{G}} = \mathcal{G}e^{yH}\mathcal{G}_{\text{br}},\tag{12}$$

where coordinate $y \in [0, 2\pi)$ parametrizes S^1 fiber and \mathcal{G}_{br} is a function of 8 Grassmann coordinates for the broken supersymmetries $v_\mu = (\theta_\mu, \bar{\theta}_\mu, \eta_\mu, \bar{\eta}_\mu)$. There have been underlined those of the $osp(4|6)$ Cartan forms that acquire dependence on dy , v and dv in addition to that on coordinates of the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset manifold.⁵ $so(8)$ generators in (11) have been transformed to the basis adapted to the Hopf fibration realization of the 7-sphere. In contrast to the consideration of Ref. [12] we manifestly decomposed $so(8)$ generators on the $SO(6)$ irreducible components and transformed them into the $\mathbf{3} \oplus \bar{\mathbf{3}}$ basis. In addition we proposed convenient realization for the Kähler 2-form on \mathbb{CP}^3 manifold so that the generators $(\tilde{V}_a{}^b, T_a, T^a)$ span its $su(4)$ isometry algebra and commute with the S^1 generator H [19], [28], [25]. Remaining 12 generators $(\tilde{T}_a, \tilde{T}^a, V_a{}^4, V_4{}^a)$ belong to the $so(8)/(su(4) \times u(1))$ coset. This choice of the Kähler tensor also diagonalizes two projectors [16], [12] that divide 32 fermionic generators of $osp(4|8)$ superalgebra (and associated coordinates) into 24 generators of $osp(4|6)$ superalgebra and 8 generators corresponding to the supersymmetries broken by the $AdS_4 \times \mathbb{CP}^3$ superbackground. In conformal basis these generators are $(Q_\mu^\mu, \bar{Q}_{\mu a}; S^{\mu a}, \bar{S}_a^\mu)$ and $(Q_\mu^4, \bar{Q}_{\mu 4}; S^{\mu 4}, \bar{S}_4^\mu)$ respectively in accordance with the decomposition $\mathbf{4} = \mathbf{3} \oplus \mathbf{1}$, $\bar{\mathbf{4}} = \bar{\mathbf{3}} \oplus \bar{\mathbf{1}}$ of the (anti)fundamental representation of $SU(4)$ on $SU(3)$ representations.

As a result the $AdS_4 \times S^7$ supervielbein bosonic components have the following expression in terms of $osp(4|8)$ Cartan forms

$$\begin{aligned}\hat{E}^{m'}(d) = & \left(\frac{1}{2}(\underline{\omega}^m(d) + \underline{c}^m(d)), -\underline{\Delta}(d)\right), \quad \hat{E}^{11}(d) = h(d) + \tilde{\Omega}_a{}^a(d) \\ E_a(d) = & i(\Omega_a(d) + \tilde{\Omega}_a(d)), \quad E^a(d) = i(\Omega^a(d) + \tilde{\Omega}^a(d))\end{aligned}\tag{13}$$

generalizing (8). To perform the reduction to 10 dimensions it is necessary to single out dy -dependent contributions in (13). Because of the form of (anti)commutation relations of $osp(4|8)$ superalgebra such terms appear only in the supervielbein components tangent to Anti-de Sitter part of the background and S^1

$$\hat{E}^{m'}(d) = G^{m'}(d) + G_y^{m'}dy, \quad \hat{E}^{11}(d) = \Phi dy + a(d).\tag{14}$$

Since $G_y^{m'} \neq 0$ the form of $\hat{E}^{m'}(d)$ deviates from the Kaluza-Klein ansatz [13], [14] so that the $SO(1,4)$ tangent space Lorentz rotation should be applied to remove the contributions

⁵Note that such a choice of the $OSp(4|8)/(SO(1,3) \times SO(7))$ element rules out dependence of the $osp(4|8)$ Cartan forms on y itself.

proportional to dy

$$\begin{aligned}(\mathbb{L}\hat{E})^{m'}(d) &= \mathbb{L}^{m'}_{n'}\hat{E}^{n'} + \mathbb{L}^{m'}_{11}\hat{E}^{11} = E^{m'}(d), \\ (\mathbb{L}\hat{E})^{11}(d) &= \mathbb{L}^{11}_{m'}\hat{E}^{m'} + \mathbb{L}^{11}_{11}\hat{E}^{11} = \Phi_L(dy + A_L(d)).\end{aligned}\tag{15}$$

$E^{m'}(d)$ is identified with the $AdS_4 \times \mathbb{CP}^3$ supervielbein components tangent to AdS_4 space-time,

$$\Phi_L = \sqrt{\Phi^2 + G_y^2}, \quad G_y^2 = G_{ym'}G_y^{m'} = G_y \cdot G_y \tag{16}$$

determines the $D = 10$ dilaton superfield $\phi(v) = 3/2 \log \Phi_L$ and A_L – RR 1-form potential. The components of the Lorentz rotation matrix

$$||\mathbb{L}|| = \begin{pmatrix} \mathbb{L}^{m'}_{n'} & \mathbb{L}^{m'}_{11} \\ \mathbb{L}^{11}_{m'} & \mathbb{L}^{11}_{11} \end{pmatrix} \in SO(1, 4), \tag{17}$$

defined by the condition $\mathbb{L}^{m'}_{n'}G_y^{n'} + \mathbb{L}^{m'}_{11}\Phi = 0$ read

$$\begin{aligned}\mathbb{L}^{m'}_{n'} &= \delta^{m'}_{n'} + \frac{\Phi - \Phi_L}{\Phi_L G_y^2} G_y^{m'} G_{yn'}, \quad \mathbb{L}^{m'}_{11} = -\Phi_L^{-1} G_y^{m'}, \\ \mathbb{L}^{11}_{m'} &= \Phi_L^{-1} G_{ym'}, \quad \mathbb{L}^{11}_{11} = \Phi_L^{-1} \Phi.\end{aligned}\tag{18}$$

Other $AdS_4 \times S^7$ supervielbein bosonic components $E_a(d)$ and $E^a(d)$ do not depend on dy and can be directly identified with the $AdS_4 \times \mathbb{CP}^3$ supervielbein components tangent to the \mathbb{CP}^3 manifold.

3 Massless $AdS_4 \times \mathbb{CP}^3$ superparticle

Massless $AdS_4 \times \mathbb{CP}^3$ superparticle action arises in the tension-to-infinity limit of the $AdS_4 \times \mathbb{CP}^3$ superstring [12] or in the mass-to-zero limit of the $D0$ –brane [18] and is constructed out of the world-line pullbacks of supervielbein bosonic components discussed in the previous Section

$$S = \int \frac{d\tau}{e} \Phi_L \left(E_{\tau m'} E_\tau^{m'} - E_{\tau a} E_\tau^a \right). \tag{19}$$

Note that when the coordinates related to the generators of broken supersymmetries are set to zero by using the κ –symmetry above action functional reduces to that of the massless superparticle on the $OSp(4|6)/(SO(1, 3) \times U(3))$ supermanifold [10]. Substituting expressions (14), (15), (18) that define the form of the supervielbein components tangent to AdS_4 allows to rewrite (19) as

$$S = \int \frac{d\tau}{e} \Phi_L \left(G_{\tau m'} G_\tau^{m'} - \Phi_L^{-2} [(G_\tau \cdot G_y)^2 - G_y^2 a_\tau^2 + 2\Phi a_\tau (G_\tau \cdot G_y)] - E_{\tau a} E_\tau^a \right). \tag{20}$$

Since the mass-shell condition following upon the action variation on the Lagrange multiplier $e(\tau)$ is irrelevant to establishing integrability of other equations of motion we set it to unity. Let us also note that one could redefine the Lagrange multiplier $e(\tau)$ to 'absorb' the overall factor of Φ_L . Corresponding expression for the Lax pair component \mathcal{L} follows by putting $\Phi_L = 1$ in (31)–(33).

As the Lax pair encoding the $OSp(4|6)/(SO(1, 3) \times U(3))$ superparticle equations is expressed in terms of the $osp(4|6)$ Cartan forms [26] it is helpful to expand $G^{m'}(d)$ on them

and dv

$$\begin{aligned}
G^{m'}(d) &= G^{0'n}(d)M_n^{m'} + G^{3n}(d)N_n^{m'} + \Delta(d)L^{m'} + G^{kl}(d)K_{kl}^{m'} \\
&+ q^{m'\mu}d\theta_\mu + \bar{q}^{m'\mu}d\bar{\theta}_\mu + s^{m'\mu}d\eta_\mu + \bar{s}^{m'\mu}d\bar{\eta}_\mu, \\
E_a(d) &= i\Omega_a(d) + u_{(1)}^\mu\omega_{(1)\mu a}(d) + u_{(3)}^\mu\omega_{(3)\mu a}(d), \\
E^a(d) &= i\Omega^a(d) + \bar{u}_{(1)}^\mu\bar{\omega}_{(1)\mu}^a(d) + \bar{u}_{(3)}^\mu\bar{\omega}_{(3)\mu}^a(d).
\end{aligned} \tag{21}$$

Coefficients at the differentials of the 'broken' fermions dv are nothing but the $AdS_4 \times S^7$ supervielbein components while those at the $osp(4|6)$ Cartan forms can be named 'previelbeins'. All of them are functions of 8 fermionic coordinates for the broken supersymmetries only. Analogous expansion for $a(d)$ is

$$\begin{aligned}
a(d) &= \tilde{\Omega}_a^a(d) + G^{0'm}(d)m_m + G^{3m}(d)n_m + \Delta(d)l + G^{mn}(d)k_{mn} \\
&+ h^\mu d\theta_\mu + \bar{h}^\mu d\bar{\theta}_\mu + p^\mu d\eta_\mu + \bar{p}^\mu d\bar{\eta}_\mu.
\end{aligned} \tag{22}$$

Coefficients h^μ, p^μ and c.c. can be identified with the corresponding $AdS_4 \times S^7$ supervielbein components and similarly m_m, n_m, l and k_{mn} can be considered as 'previelbeins'.

For practical calculations explicit form of the above introduced expansion coefficients is required that can be derived upon specifying \mathcal{G}_{br} in (12). We concentrate on the same representative used in our previous studies [19], [25]⁶

$$\mathcal{G}_{br} = e^{\theta^\mu Q_\mu + \bar{\theta}^\mu \bar{Q}_\mu} e^{\eta_\mu S^{\mu 4} + \bar{\eta}_\mu \bar{S}_4^\mu}. \tag{23}$$

Then the 'previelbein' coefficients in (21) take the form

$$\begin{aligned}
M_n^m &= \delta_n^m \left[1 - (\theta\bar{\theta})(\eta\bar{\eta}) + \frac{1}{4}(\theta^2\bar{\theta}^2 + \eta^2\bar{\eta}^2) + \frac{1}{8}\theta^2\bar{\theta}^2\eta^2\bar{\eta}^2 \right] - i(\theta\sigma_n\tilde{\sigma}^m\bar{\eta} + \bar{\theta}\sigma_n\tilde{\sigma}^m\eta) \\
&+ 2 \left\{ 1 - \frac{i}{2}[(\theta\bar{\eta}) + (\bar{\theta}\eta)] \right\} (\theta\sigma_n\bar{\theta})(\eta\sigma^m\bar{\eta}), \quad M_n^3 = [(\theta\bar{\eta}) - (\bar{\theta}\eta)](\theta\sigma_n\bar{\theta}), \\
N_n^m &= \delta_n^m \left[-(\theta\bar{\theta})(\eta\bar{\eta}) + \frac{1}{4}(\theta^2\bar{\theta}^2 - \eta^2\bar{\eta}^2) + \frac{1}{8}\theta^2\bar{\theta}^2\eta^2\bar{\eta}^2 \right] - i(\theta\sigma_n\tilde{\sigma}^m\bar{\eta} + \bar{\theta}\sigma_n\tilde{\sigma}^m\eta) \\
&+ 2 \left\{ 1 - \frac{i}{2}[(\theta\bar{\eta}) + (\bar{\theta}\eta)] \right\} (\theta\sigma_n\bar{\theta})(\eta\sigma^m\bar{\eta}), \quad N_n^3 = [(\theta\bar{\eta}) - (\bar{\theta}\eta)](\theta\sigma_n\bar{\theta}), \\
L^m &= [(\bar{\theta}\eta) - (\theta\bar{\eta})](\eta\sigma^m\bar{\eta}), \quad -L^3 = 1 + i[(\theta\bar{\eta}) + (\bar{\theta}\eta)], \\
K_{kl}^m &= -\frac{i}{2}\varepsilon_{kl}^m \left\{ \left(1 + \frac{1}{2}\eta^2\bar{\eta}^2 \right) (\theta\bar{\theta}) + (\eta\bar{\eta}) \right\} + \frac{1}{2}[(\bar{\theta}\sigma_{kl}\eta) - (\theta\sigma_{kl}\bar{\eta})](\eta\sigma^m\bar{\eta}), \\
K_{kl}^3 &= -\frac{i}{2}[(\bar{\theta}\sigma_{kl}\eta) + (\theta\sigma_{kl}\bar{\eta})]
\end{aligned} \tag{24}$$

and

$$u_{(1,3)}^\mu = \mp 2\{\theta^\mu \pm i\eta^\mu[1 \pm (\theta\bar{\theta})]\}, \quad \bar{u}_{(1,3)}^\mu = \mp 2\{\bar{\theta}^\mu \mp i\bar{\eta}^\mu[1 \mp (\theta\bar{\theta})]\}, \tag{25}$$

and those from (22) read

$$\begin{aligned}
m_m &= \left\{ 1 - i[(\theta\bar{\eta}) + (\bar{\theta}\eta)] \right\} (\theta\sigma_m\bar{\theta}) + \left(1 - \frac{1}{2}\theta^2\bar{\theta}^2 \right) (\eta\sigma_m\bar{\eta}), \\
n_m &= \left\{ 1 - i[(\theta\bar{\eta}) + (\bar{\theta}\eta)] \right\} (\theta\sigma_m\bar{\theta}) - \left(1 + \frac{1}{2}\theta^2\bar{\theta}^2 \right) (\eta\sigma_m\bar{\eta}) \\
l &= (\bar{\theta}\eta) - (\theta\bar{\eta}), \quad k_{mn} = \frac{1}{2}[(\bar{\theta}\sigma_{mn}\eta) - (\theta\sigma_{mn}\bar{\eta})] - i(\theta\bar{\theta})(\eta\sigma_{mn}\bar{\eta}).
\end{aligned} \tag{26}$$

Non-zero $AdS_4 \times S^7$ supervielbein components that enter (21) and (22) are given by

$$\begin{aligned}
q^{m\mu} &= \frac{i}{2} \left(1 + \frac{1}{2}\eta^2\bar{\eta}^2 \right) \tilde{\sigma}^{m\mu\nu}\bar{\theta}_\nu + \frac{1}{2}\bar{\eta}^2\tilde{\sigma}^{m\mu\nu}\eta_\nu, \quad q^{3\mu} = -i\bar{\eta}^\mu, \\
s^{m\mu} &= \frac{i}{2}\tilde{\sigma}^{m\mu\nu}\bar{\eta}_\nu, \quad h^\mu = -\bar{\eta}^\mu + i(\bar{\theta}\eta)\bar{\eta}^\mu - i(\bar{\theta}\bar{\eta})\eta^\mu
\end{aligned} \tag{27}$$

⁶Let us note that fermionic coordinates associated with the generators of superconformal algebras were introduced in [29], [30], [31], [32] to obtain explicit form of the Lagrangians of string/brane models related to the maximally supersymmetric instances of AdS/CFT correspondence [3].

and c.c. expressions. Finally proportional to dy contributions to the $AdS_4 \times S^7$ supervielbein components in directions tangent to Anti-de Sitter space-time and S^1 fiber acquire the form

$$\begin{aligned} G_y^m &= 2 \left(1 + \frac{1}{2} \eta^2 \bar{\eta}^2 \right) (\theta \sigma^m \bar{\theta}) + 2 \{ 1 - i[(\theta \bar{\eta}) + (\bar{\theta} \eta)] \} (\eta \sigma^m \bar{\eta}), \quad G_y^3 = 2[(\theta \bar{\eta}) - (\bar{\theta} \eta)], \\ \Phi &= 1 - 2i[(\theta \bar{\eta}) + (\bar{\theta} \eta)] + 4[(\theta \eta)(\bar{\theta} \bar{\eta}) - (\theta \bar{\eta})(\bar{\theta} \eta)]. \end{aligned} \quad (28)$$

Superparticle equations of motion can be written in the form facilitating their Lax representation. Taking $osp(4|6)/(so(1,3) \times u(3))$ Cartan forms (4) as independent variation parameters yields the set of bosonic and fermionic equations of motion

$$\begin{aligned} -\frac{\delta S}{\delta G_{\tau}^{0'm}(\delta)} &= \dot{a}^{0'm} + 2G_{\tau}^m a^{0'n} - 4l^{mn} G_{\tau}^{0'n} + 4f G_{\tau}^{3m} - 2\Delta_{\tau} b^{3m} \\ &\quad - 4i \left(\omega_{(1)\tau a} \sigma^m \bar{\varepsilon}_{(1)}^a - \varepsilon_{(1)a} \sigma^m \bar{\omega}_{(1)\tau}^a + \omega_{(3)\tau a} \sigma^m \bar{\varepsilon}_{(3)}^a - \varepsilon_{(3)a} \sigma^m \bar{\omega}_{(3)\tau}^a \right) = 0, \\ -\frac{1}{2} \frac{\delta S}{\delta \Delta(\delta)} &= \dot{f} + G_{\tau}^{0'm} b^{3m} - G_{\tau 3m} a^{0'm} \\ &\quad - 2 \left(\omega_{(1)\tau a}^{\mu} \bar{\varepsilon}_{(1)\mu}^a - \varepsilon_{(1)a}^{\mu} \bar{\omega}_{(1)\tau\mu}^a - \omega_{(3)\tau a}^{\mu} \bar{\varepsilon}_{(3)\mu}^a + \varepsilon_{(3)a}^{\mu} \bar{\omega}_{(3)\tau\mu}^a \right) = 0, \\ -\frac{\delta S}{\delta \Omega_a(\delta)} &= \dot{y}^a + i y^b \left(\tilde{\Omega}_{\tau b}^a + \delta_b^a \tilde{\Omega}_{\tau c}^c \right) - 4i w \Omega_{\tau}^a \\ &\quad + 4i \varepsilon^{abc} \left(\omega_{(1)\tau b}^{\mu} \varepsilon_{(1)\mu c} - \omega_{(3)\tau b}^{\mu} \varepsilon_{(3)\mu c} \right) = 0 \end{aligned} \quad (29)$$

and

$$\begin{aligned} \frac{1}{4} \frac{\delta S}{\delta \bar{\omega}_{(1)\mu}^a(\delta)} &= \dot{\varepsilon}_{(3)a}^{\mu} + \frac{1}{2} \left(G_{\tau}^{mn} \varepsilon_{(3)a}^{\nu} - l^{mn} \omega_{(3)\tau a}^{\nu} \right) \sigma_{mn\nu}^{\mu} + i \tilde{\sigma}_m^{\mu\nu} \left(G_{\tau}^{3m} \varepsilon_{(3)\nu a} - \frac{1}{2} b^{3m} \omega_{(3)\tau\nu a} \right) \\ &\quad + i \tilde{\sigma}_m^{\mu\nu} \left(G_{\tau}^{0'm} \varepsilon_{(1)\nu a} - \frac{1}{2} a^{0'm} \omega_{(1)\tau\nu a} \right) + \Delta_{\tau} \varepsilon_{(1)a}^{\mu} - f \omega_{(1)\tau a}^{\mu} \\ &\quad - i \left(\tilde{\Omega}_{\tau a}^b - \delta_a^b \tilde{\Omega}_{\tau c}^c \right) \varepsilon_{(3)b}^{\mu} - 2i w \omega_{(3)\tau a}^{\mu} - i \varepsilon_{abc} \left(\Omega_{\tau}^b \bar{\varepsilon}_{(1)}^{\mu c} - y^b \bar{\omega}_{(1)\tau}^{\mu c} \right) = 0, \\ -\frac{1}{4} \frac{\delta S}{\delta \bar{\omega}_{(3)\mu}^a(\delta)} &= \dot{\varepsilon}_{(1)a}^{\mu} + \frac{1}{2} \left(G_{\tau}^{mn} \varepsilon_{(1)a}^{\nu} - l^{mn} \omega_{(1)\tau a}^{\nu} \right) \sigma_{mn\nu}^{\mu} - i \tilde{\sigma}_m^{\mu\nu} \left(G_{\tau}^{3m} \varepsilon_{(1)\nu a} - \frac{1}{2} b^{3m} \omega_{(1)\tau\nu a} \right) \\ &\quad - i \tilde{\sigma}_m^{\mu\nu} \left(G_{\tau}^{0'm} \varepsilon_{(3)\nu a} - \frac{1}{2} a^{0'm} \omega_{(3)\tau\nu a} \right) + \Delta_{\tau} \varepsilon_{(3)a}^{\mu} - f \omega_{(3)\tau a}^{\mu} \\ &\quad - i \left(\tilde{\Omega}_{\tau a}^b - \delta_a^b \tilde{\Omega}_{\tau c}^c \right) \varepsilon_{(1)b}^{\mu} - 2i w \omega_{(1)\tau a}^{\mu} - i \varepsilon_{abc} \left(\Omega_{\tau}^b \bar{\varepsilon}_{(3)}^{\mu c} - y^b \bar{\omega}_{(3)\tau}^{\mu c} \right) = 0 \end{aligned} \quad (30)$$

and c.c. equations. In (29) and (30) we have introduced the following bosonic and fermionic quantities

$$\begin{pmatrix} a^{0'm} \\ 2f \\ -b_{3m} \\ -l_{kl} \end{pmatrix} = 2 \begin{pmatrix} M_m^{n'} \\ L^{n'} \\ N_m^{n'} \\ K_{kl}^{n'} \end{pmatrix} [\Phi_L G_{\tau n'} - \Phi_L^{-1} (G_{\tau} \cdot G_y + \Phi a_{\tau}) G_{y n'}] + 2 \Phi_L^{-1} \begin{pmatrix} m_m \\ l \\ n_m \\ k_{kl} \end{pmatrix} (G_y^2 a_{\tau} - \Phi G_{\tau} \cdot G_y), \quad (31)$$

$$y^a = -i \Phi_L E_{\tau}^a, \quad \bar{y}_a = -i \Phi_L E_{\tau a}, \quad w = \frac{1}{2} \Phi_L^{-1} (G_y^2 a_{\tau} - \Phi G_{\tau} \cdot G_y). \quad (32)$$

and

$$\begin{aligned} \varepsilon_{(1)a}^{\mu} &= -\frac{1}{4} \Phi_L E_{\tau a} \bar{u}_{(3)}^{\mu}, \quad \varepsilon_{(3)a}^{\mu} = \frac{1}{4} \Phi_L E_{\tau a} \bar{u}_{(1)}^{\mu}, \\ \bar{\varepsilon}_{(1)}^{\mu a} &= \frac{1}{4} \Phi_L E_{\tau}^a u_{(3)}^{\mu}, \quad \bar{\varepsilon}_{(3)}^{\mu a} = -\frac{1}{4} \Phi_L E_{\tau}^a u_{(1)}^{\mu} \end{aligned} \quad (33)$$

that enter the Lax component \mathcal{L} (35). In addition there are 8 equations stemming from the

variation of the action (19) on v . They can be brought to the following form

$$\begin{aligned}
& -2 \frac{d}{d\tau} \left[\Phi_L G_{\tau m'} \frac{\partial G_{\tau}^{m'}}{\partial \dot{v}_\mu} - \Phi_L^{-1} (G_\tau \cdot G_y + \Phi a_\tau) G_{ym'} \frac{\partial G_{\tau}^{m'}}{\partial \dot{v}_\mu} + \Phi_L^{-1} (G_y^2 a_\tau - \Phi G_\tau \cdot G_y) \frac{\partial a_\tau}{\partial \dot{v}_\mu} \right] \\
& + \frac{\partial L}{\partial \Phi_L} \frac{\partial \Phi_L}{\partial v_\mu} + \Phi_L^{-1} a_\tau \left(a_\tau \frac{\partial G_y^2}{\partial v_\mu} - 2 G_\tau \cdot G_y \frac{\partial \Phi}{\partial v_\mu} \right) + 2 \Phi_L G_{\tau m'} \left(G_\tau^{0'n} \frac{\partial M_a^{m'}}{\partial v_\mu} + G_\tau^{3n} \frac{\partial N_n^{m'}}{\partial v_\mu} \right. \\
& \quad \left. + \Delta_\tau \frac{\partial L^{m'}}{\partial v_\mu} + G_\tau^{kl} \frac{\partial K_{kl}^{m'}}{\partial v_\mu} - \dot{\theta}_\nu \frac{\partial q^{m'\nu}}{\partial v_\mu} - \dot{\bar{\theta}}_\nu \frac{\partial \bar{q}^{m'\nu}}{\partial v_\mu} - \dot{\eta}_\nu \frac{\partial s^{m'\nu}}{\partial v_\mu} - \dot{\bar{\eta}}_\nu \frac{\partial \bar{s}^{m'\nu}}{\partial v_\mu} \right) \\
& + \Phi_L \left[E_{\tau a} \left(\bar{\omega}_{(1)\tau\nu}^a \frac{\partial \bar{u}_{(1)}^\nu}{\partial v_\mu} + \bar{\omega}_{(3)\tau\nu}^a \frac{\partial \bar{u}_{(3)}^\nu}{\partial v_\mu} \right) + E_\tau^a \left(\omega_{(1)\tau\nu a} \frac{\partial u_{(1)}^\nu}{\partial v_\mu} + \omega_{(3)\tau\nu a} \frac{\partial u_{(3)}^\nu}{\partial v_\mu} \right) \right] \\
& - 2 \Phi_L^{-1} (G_\tau \cdot G_y + \Phi a_\tau) \left(G_\tau^{0'm} \frac{\partial M_{m'}^{n'} G_{yn'}}{\partial v_\mu} + G_\tau^{3m} \frac{\partial N_{m'}^{n'} G_{yn'}}{\partial v_\mu} + \Delta_\tau \frac{\partial L^{m'} G_{ym'}}{\partial v_\mu} + G_\tau^{kl} \frac{\partial K_{kl}^{n'} G_{yn'}}{\partial v_\mu} \right. \\
& \quad \left. - \dot{\theta}_\nu \frac{\partial q^{m'\nu} G_{ym'}}{\partial v_\mu} - \dot{\bar{\theta}}_\nu \frac{\partial \bar{q}^{m'\nu} G_{ym'}}{\partial v_\mu} - \dot{\eta}_\nu \frac{\partial s^{m'\nu} G_{ym'}}{\partial v_\mu} - \dot{\bar{\eta}}_\nu \frac{\partial \bar{s}^{m'\nu} G_{ym'}}{\partial v_\mu} \right) \\
& + 2 \Phi_L^{-1} (G_y^2 a_\tau - \Phi G_\tau \cdot G_y) \left(G_\tau^{0'm} \frac{\partial m_m}{\partial v_\mu} + G_\tau^{3m} \frac{\partial n_m}{\partial v_\mu} + \Delta_\tau \frac{\partial l}{\partial v_\mu} + G_\tau^{mn} \frac{\partial k_{mn}}{\partial v_\mu} \right. \\
& \quad \left. - \dot{\theta}_\nu \frac{\partial h^\nu}{\partial v_\mu} - \dot{\bar{\theta}}_\nu \frac{\partial \bar{h}^\nu}{\partial v_\mu} - \dot{\eta}_\nu \frac{\partial p^\nu}{\partial v_\mu} - \dot{\bar{\eta}}_\nu \frac{\partial \bar{p}^\nu}{\partial v_\mu} \right) = 0,
\end{aligned} \tag{34}$$

where for the sake of uniformity of presentation we introduced the following notation $\frac{\partial G_{\tau}^{m'}}{\partial \dot{v}_\mu} = (q^{m'\mu}, \bar{q}^{m'\mu}, s^{m'\mu}, \bar{s}^{m'\mu})$ and $\frac{\partial a_\tau}{\partial \dot{v}_\mu} = (h^\mu, \bar{h}^\mu, p^\mu, \bar{p}^\mu)$.⁷

Fulfilment of the $AdS_4 \times \mathbb{CP}^3$ superparticle equations of motion (29), (30), (34) is equivalent to the Lax equation (1) with the Lax component \mathcal{M} given by the world-line projection of the $osp(4|6)$ Cartan forms (4) and another component can be presented as

$$\mathcal{L} = \mathcal{L}_{so(2,3)} + \mathcal{L}_{su(4)} + \mathcal{L}_{24susys} \in osp(4|6). \tag{35}$$

The first term takes value in the $so(2,3)$ isometry algebra of AdS_4 space-time

$$\mathcal{L}_{so(2,3)} = a^{0'm} M_{0'm} + f D + b^{3m} M_{3m} + l^{mn} M_{mn}. \tag{36}$$

The second summand in (35) belongs to the $su(4)$ isometry algebra of \mathbb{CP}^3 manifold

$$\mathcal{L}_{su(4)} = y^a T_a + \bar{y}_a T^a + 4w V_a^a \tag{37}$$

and the last one is the linear combination of the fermionic generators

$$\mathcal{L}_{24susys} = \varepsilon_{(1)a}^\mu Q_{(1)\mu}^a + \bar{\varepsilon}_{(1)}^{\mu a} \bar{Q}_{(1)\mu a} + \varepsilon_{(3)a}^\mu Q_{(3)\mu}^a + \bar{\varepsilon}_{(3)}^{\mu a} \bar{Q}_{(3)\mu a}. \tag{38}$$

The Lax component \mathcal{L} can be presented in the form of $osp(4|6)$ -valued differential operator (2) acting of the superparticle action (19).

4 Conclusion

In this paper generalizing the consideration of Ref. [26] we have shown that the complete set of equations of motion for the massless $AdS_4 \times \mathbb{CP}^3$ superparticle admits Lax representation implying their classical integrability. It would be of interest to construct explicitly all the integrals of motion as well as to examine quantum integrability of this model. It is also tempting to suggest that the results reported here may be useful in proving integrability of the $AdS_4 \times \mathbb{CP}^3$ superstring and $D0$ -brane equations that can be reduced to those of the massless superparticle.

⁷We assume that the fermionic derivative acts from the right.

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References

- [1] I. Bena, J. Polchinski and R. Roiban, "Hidden symmetries of the $AdS_5 \times S^5$ superstring", Phys. Rev. **D69** (2004) 046002, [arXiv:hep-th/0305116](#).
- [2] J.A. Minahan and K. Zarembo, "The Bethe-ansatz for $\mathcal{N} = 4$ super Yang-Mills", JHEP **0303** (2003) 013, [arXiv:hep-th/0212208](#).
- [3] J.M. Maldacena, "The large N limit of superconformal field theories and supergravity", Adv. Theor. Math. Phys. **2** (1998) 231, [arXiv:hep-th/9711200](#).
S.S. Gubser, I.R. Klebanov and A.M. Polyakov, "Gauge theory correlators from non-critical string theory", Phys. Lett. **B428** (1998) 105, [arXiv:hep-th/9802109](#).
E. Witten, "Anti-de Sitter space and holography", Adv. Theor. Math. Phys. **2** (1998) 253, [arXiv:hep-th/9802150](#).
- [4] N. Beisert et. al., "Review of AdS/CFT Integrability: An Overview", Lett. Math. Phys. **99** (2012) 3, [arXiv:1012.3982 \[hep-th\]](#).
- [5] O. Aharony, O. Bergman, D.L. Jafferis and J. Maldacena, " $\mathcal{N} = 6$ superconformal Chern-Simons-matter theories, M2-branes and their gravity duals", JHEP **0810** (2008) 091, [arXiv:0806.1218 \[hep-th\]](#).
- [6] S. Watamura, "Spontaneous compactification and $Cp(N)$: $SU(3) \times SU(2) \times U(1)$, $\sin^2 \theta_W$, $g(3)/g(2)$ and $SU(3)$ triplet chiral fermions in 4 dimensions", Phys. Lett. **B136** (1984) 245.
- [7] T. Klose, "Review of AdS/CFT integrability, Chapter IV.3: $\mathcal{N} = 6$ Chern-Simons and strings on $AdS_4 \times CP^3$ ", [arXiv:1012.3999 \[hep-th\]](#).
- [8] R.R. Metsaev and A.A. Tseytlin, "Type IIB superstring action in $AdS_5 \times S^5$ background", Nucl. Phys. **B533** (1998) 109, [arXiv:hep-th/9805028](#).
- [9] G. Arutyunov and S. Frolov, "Superstrings on $AdS_4 \times CP^3$ as a Coset Sigma-model", JHEP **0809** (2008) 129, [arXiv:0806.4940 \[hep-th\]](#).
- [10] B.J. Stefanski, "Green-Schwarz action for Type IIA strings on $AdS_4 \times CP^3$ ", Nucl. Phys. **B808** (2009) 80, [arXiv:0806.4948 \[hep-th\]](#).
- [11] P. Fre and P.A. Grassi, "Pure Spinor Formalism for $OSp(N|4)$ backgrounds", [arXiv:0807.0044 \[hep-th\]](#).
G. Bonelli, P.A. Grassi and H. Safaai, "Exploring pure spinor string theory on $AdS_4 \times \mathbb{CP}^3$ ", JHEP **0810** (2008) 085, [arXiv:0808.1051 \[hep-th\]](#).
R. D'Auria, P. Fre, P.A. Grassi and M. Trigiante, "Superstrings on $AdS_4 \times CP^3$ from Supergravity", Phys. Rev. **D79** (2009) 086001, [arXiv:0808.1282 \[hep-th\]](#).
- [12] J. Gomis, D. Sorokin and L. Wulff, "The complete $AdS_4 \times CP^3$ superspace for type IIA superstring and D -branes", JHEP **0903** (2009) 015, [arXiv:0811.1566 \[hep-th\]](#).

- [13] M.J. Duff, P.S. Howe, T. Inami and K.S. Stelle, "Superstrings in $D = 10$ from supermembranes in $D = 11$ ", Phys. Lett. **B191** (1987) 70.
- [14] P.S. Howe and E. Sezgin, "The supermembrane revisited", Class. Quantum Grav. **22** (2005) 2167, [arXiv:hep-th/0412245](#).
- [15] B. de Wit, K. Peeters, J. Plefka and A. Sevrin, "The M-theory two-brane in $AdS_4 \times S^7$ and $AdS_7 \times S^4$ ", Phys. Lett. **B443** (1998) 153, [arXiv:hep-th/9808052](#).
- [16] B.E.W. Nilsson and C. Pope, "Hopf fibration of eleven dimensional supergravity", Class. Quantum Grav. **1** (1984) 499.
- [17] D.P. Sorokin, V.I. Tkach and D.V. Volkov, "Kaluza-Klein theories and spontaneous compactification mechanisms of extra space dimensions", In **Moscow 1984, Proceedings, Quantum Gravity**, 376-392.
D.P. Sorokin, V.I. Tkach and D.V. Volkov, "On the relationship between compactified vacua of $D = 11$ and $D = 10$ supergravities", Phys. Lett. **B161** (1985) 301.
- [18] P.A. Grassi, D. Sorokin and L. Wulff, "Simplifying superstring and D -brane actions in $AdS_4 \times \mathbb{CP}^3$ superbackground", JHEP **0908** (2009) 060, [arXiv:0903.5407 \[hep-th\]](#).
- [19] D.V. Uvarov, " $AdS_4 \times \mathbb{CP}^3$ superstring in the light-cone gauge", Nucl. Phys. **B826** (2010) 294, [arXiv:0906.4699 \[hep-th\]](#).
D.V. Uvarov, "Light-cone gauge Hamiltonian for $AdS_4 \times \mathbb{CP}^3$ superstring", Mod. Phys. Lett. **A25** (2010) 1251, [arXiv:0912.1044 \[hep-th\]](#).
- [20] I. Adam, A. Dekel, L. Mazzucato and Y. Oz, "Integrability of Type II superstrings on Ramond-Ramond backgrounds in various dimensions", JHEP **0706** (2007) 085, [arXiv:hep-th/0702083](#).
- [21] A. Babichenko, B. Stefanski and K. Zarembo, "Integrability and the AdS_3/CFT_2 correspondence", JHEP **1003** (2010) 058, [arXiv:0912.1723 \[hep-th\]](#).
- [22] D. Sorokin, A. Tseytlin, L. Wulff and K. Zarembo, "Superstrings in $AdS_2 \times S^2 \times T^6$ ", J. Phys. **A44** (2011) 275401, [arXiv:1104.1793 \[hep-th\]](#).
- [23] D. Sorokin and L. Wulff, "Evidence for the classical integrability of the complete $AdS_4 \times \mathbb{CP}^3$ superstring", JHEP **1011** (2010) 143, [arXiv:1009.3498 \[hep-th\]](#).
- [24] A. Cagnazzo, D. Sorokin and L. Wulff, "More on integrable structures of superstrings in $AdS_4 \times \mathbb{CP}^3$ and $AdS_2 \times S^2 \times T_6$ superbackgrounds", JHEP **1201** (2012) 004, [arXiv:1111.4197 \[hep-th\]](#).
- [25] D.V. Uvarov, "Kaluza-Klein gauge and minimal integrable extension of $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model", Int. J. Mod. Phys. **A27** (2012) 1250118, [arXiv:1203.3041 \[hep-th\]](#).
- [26] D.V. Uvarov, "Lagrangian mechanics of massless superparticle on $AdS_4 \times \mathbb{CP}^3$ superbackground", Nucl. Phys. **B867** (2013) 354, [arXiv:1205.5388 \[hep-th\]](#).
- [27] D.V. Uvarov, " $AdS_4 \times \mathbb{CP}^3$ superstring and $D = 3$ $\mathcal{N} = 6$ superconformal symmetry", Phys. Rev. **D79** (2009) 106007, [arXiv:0811.2813 \[hep-th\]](#).

- [28] D.V. Uvarov, " $D = 3$ $\mathcal{N} = 6$ superconformal symmetry of $AdS_4 \times \mathbb{CP}^3$ superstring", Class. Quantum Grav. **28** (2011) 235010, [arXiv:1011.5457 \[hep-th\]](#).
- [29] G. Dall'Agata, D. Fabbri, C. Fraser, P. Fre, P. Termonia and M. Trigiante, "The $OSp(8|4)$ singleton action from the supermembrane", Nucl. Phys. **B542** (1999) 157, [arXiv:hep-th/9807115](#).
- [30] R. Kallosh, "Superconformal actions in Killing gauge", [arXiv:hep-th/9807206](#).
- [31] P. Pasti, D.P. Sorokin and M. Tonin, "On gauge-fixed superbrane actions in AdS superbackgrounds", Phys. Lett. **B447** (1999) 251, [arXiv:hep-th/9809213](#).
- [32] R.R. Metsaev and A.A. Tseytlin, "Superstring action in $AdS_5 \times S^5$: κ -symmetry light cone gauge", Phys. Rev. **D63** (2001) 046002, [arXiv:hep-th/0007036](#).
R.R. Metsaev, C.B. Thorn and A.A. Tseytlin, "Light-cone superstring in AdS space-time", Nucl. Phys. **B596** (2001) 151, [arXiv:hep-th/0009171](#).